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Departamento de Economía de la Empresa
Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)
Fax (34) 91 624-9608

BANK RUNS, SUSPENSION OF CONVERTIBILIY VERSUS DEPOSIT INSURANCE

Margarita Samartín*

Abstract

This paper models information-induced and "pure-panic" runs in the banking system, in an environment of risk-averse agents. In this framework, deposits are needed to provide insurance against investors' unexpected demand for liquidity and therefore, a role for a financial intermediary is justified. Conditions to assure bank-runs as an equilibrium phenomenon are derived, and a welfare analysis of two devices that have traditionally been used by banks in order to prevent runs (namely, suspension of convertibility versus deposit insurance), is presented. The analysis shown in this paper finds support for the "narrow banking" proposal that has been currently discussed in the literature.

Key Words and Phrases
Banking, Deposit Contracts, Deposit Insurance, Suspension of convertibility.

*Samartín, Departamento de Economía de la Empresa de la Universidad Carlos III de Madrid.
email: samartin@emp.uc3m.es
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1.- Introduction

Until the 1930s, the US financial system was hampered by banking panics. The social and economic importance of the phenomenon attracted considerable research effort (e.g., Gurley and Shaw [18], Tobin [25], Friedman and Schwartz [15], Stiglitz [24]), which aimed at explaining why panics occurred.

Recent research of the banking theory, however, has provided additional insights into the underlying reasons of panics, especially by examining the economic function of deposit contracts (e.g. Diamond and Dybvig [12], Bhattacharya and Gale [3], Bhattacharya and Jacklin [2] and Chari and Jagannathan [9]). This latter stream of research has approached banking panics through two different types of models.

First, the models of pure panic runs comprise those models in which bank runs occur as random phenomena, with no correlation with other economic variables. Diamond and Dybvig [12] made a significant contribution by introducing into the model the demand for liquidity and the transformation service provided by banks. They demonstrated that demand deposit contracts, which enable the transformation of illiquid assets into more liquid liabilities, provide a rationale both for the existence of banks and for their vulnerability to runs. The optimal contract yields a higher level of consumption for those who withdraw early than the technological return. Bank runs, thus, take place when the idea of deposit withdrawals spills over economic agents (an essential point is that banks satisfy a sequential service constraint). The model states that under no aggregate uncertainty, a suspension of convertibility policy can hinder the bank run equilibrium. Otherwise, a deposit insurance policy would be more effective. Diamond and Dybvig's model attracted severe criticisms (e.g., Gorton [17]) for assuming that bank runs are random phenomena, and thus, uncorrelated with other economic variables. Gorton [17], in an empirical study of bank runs in the US during the National Banking Era (1863-1913), found support for the notion that bank runs tended to occur after business cycle peaks. Bhattacharya and Gale [3] considered a variation of the Diamond and Dybvig's model with many intermediaries subjected to privately observed liquidity shocks. They demonstrated that unconstrained walrasian trading among intermediaries would lead to underinvestment in liquid assets. Moreover, their model showed the welfare gains from setting up an institution such as a central bank, offering borrowing and lending opportunities at a subsidized rate.

Second, models of information-induced runs assert that bank runs occur due to the diffusion of negative information among depositors about bank's solvency (e.g., Bhattacharya and Jacklin [2] and Chari and

Jagannathan [9]). Bhattacharya and Jacklin [2] examined the relative degrees of risk-sharing provided by bank deposit and traded equity contracts. They focused on the relationship between riskness and information of the stream of returns and the desirability of equity over deposit contracts. They found that deposit contracts tended to the better for financing low risk assets. Chari and Jagannathan [9] drew on both information-induced and pure panic runs models[1] to study the effects of extra market constraints, such as suspension of convertibility on bank runs. They concluded that such constraints prevent bank runs and result in superior allocations. Despite the importance of this contribution, it raised considerable criticisms due for: i) the ambiguous role of banks or any other financial intermediary in the model, ii) being assumed that individuals were risk neutral.

This paper intends to fill several research gaps of the recent theory on banking panics. First, this study introduces risk-averse preferences in Chari and Jagannathan's model. The importance of this extension rests on the distinctive role of financial intermediaries in the economy, especially by providing insurance to individuals subject to preference shocks. Second, this paper examines the optimal intervention mechanism from a public finance perspective: the choice between suspension of convertibility or deposit insurance, given their relative benefits and costs (of randomization in meeting liquidity needs or deadweight taxation). In this framework, suspension does no longer lead to an improvement of ex-ante utility and is not always the preferred instrument to cope with runs. The level of risk-aversion and the dispersion of the random asset, measured by the standard deviation, are crucial variables to the analysis. The numerical simulations shown in this paper suggest that deposit insurance should only be restricted to finance low risk assets. In this sense, the results of this paper support the "narrow banking" idea that has been currently discussed in the literature.

The structure of the paper is as follows: The basic framework of the model is presented in section 2, the ex-ante (banker's) contract and the ex-post (depositors') problems are defined in sections 2.1 and 2.2 respectively. A condition to assure a panic run is derived in section 3. Section 4 analyzes two traditional devices in order to prevent runs and a welfare comparison of both measures is presented in section 5. Section 6 ends the paper with some conclusions with respect to the above public measures.

2.- The model

The model can be summarized as follows:

(i) Hypotheses

(a) Three period economy, T=0,1,2

(b) A single commodity.

(c) Investment technology: There are two investment technologies on the side of the intermediary:

- A short-term asset at T=0 that yields one unit at T=1
- A long-term asset at T=0 that yields a random return $\tilde{R}$ at T=2. For simplicity, it is assumed that the long-term technology cannot be liquidated early (or equivalently, only at a loss)[2].

The random return is defined as follows:

$$\tilde{R} \in (Rh, Rl) \quad \text{with} \quad p.d.f\ (1-p, p) \quad \text{and} \quad Rl \to 0 \qquad [1]$$

- It is also assumed that the probability of the low return occurring (p), is sufficiently small. As it will be seen later on, this assumption allows to simplify the ex-ante contract.

(d) Preferences: There is a continuum of ex-ante identical agents at T=0 that maximize expected utility of consumption. They are subject at T=1 to a privately observed uninsurable risk of being of either of two types.

- Type-1 agents derive only utility from consumption in period one.
- Type-2 agents derive utility from consumption in both periods 1 and 2, i.e:

$$\begin{aligned} &\text{Type-1 agents} \quad U^1(c_1,c_2,\rho_1) = \rho_1 U(c_1) + (1-\rho_1)U(c_2) \\ &\text{Type-2 agents} \quad U^2(c_1,c_2,\rho_2) = \rho_2 U(c_1) + (1-\rho_2)U(c_2) \end{aligned} \qquad [2]$$

It is assumed that $0 \leq \rho_2 < \rho_1 \leq 1$ and $\rho_1 \to 1$.

The proportion of type-1 agents is stochastic and defined as:

$$\tilde{t} \in (t_1, t_2) \quad \text{with} \quad p.d.f\ (r_1, r_2) \quad \text{and} \quad \begin{aligned} &r_1 + r_2 = 1 \\ &t_1 < t_2 \end{aligned} \qquad [3]$$

(e) Initial endowments: All agents are endowed with one unit of the good at T=0.

(f) Information: At T=1 a random fraction, $\tilde{\alpha}$, of type-2 individuals receives information about time 2 returns[3].

It is assumed that this information is perfect.

The random variable $\tilde{\alpha}$, is defined as follows:

$$\tilde{\alpha} \in (0, \alpha) \quad \text{with} \quad p.d.f\ (1-q, q) \tag{4}$$

It is observed that in some states of nature, there will be no informed agents in the model.

As in Chari and Jagannathan [9], the random proportion of type-1 agents is needed in order to create confusion between a large withdrawal queue size at the bank due to liquidity shocks, t_2 realized, or negative information shocks.

(g) Parameter restrictions: In order for individuals to have a non-trivial signalling extraction problem, the following parameter restriction is assumed, (it will become clear later why this assumption is needed).

$$t_2 = t_1 + \alpha(1 - t_1) \tag{5}$$

(h) Utility functions: Considering the agents' preferences hypothesis and in order to get numerical results, the following form for the utility function is assumed:

$$U^i(c_1, c_2, \rho_i) = \rho_i \frac{(k+c_1)^{1-\gamma}}{1-\gamma} + (1-\rho_i)\frac{(k+c_2)^{1-\gamma}}{1-\gamma} \tag{6}$$

where k is a constant and $i = 1,\ 2$.[4]

(ii) Data

The state of nature is described by the vector $\tilde{\theta} = (\tilde{t},\ \tilde{\alpha},\ \tilde{R})$ that contains the three random variables that are identical and independently distributed.

Table I shows the different states of nature and its associated probabilities.

2.1.- The banking contract: The ex-ante program.

Formally, the demand deposit contract may be defined as a contract that requires an initial investment at T=0 with the intermediary in exchange for the right to withdraw per unit of initial investment (at the discretion of depositor and conditional on the bank's solvency) either:

(i) c_{11} units in period 1 and $\tilde{c}_{21}$ units in period 2

(ii) c_{12} units in period 1 and $\tilde{c}_{22}$ units in period 2

As shown by Jacklin [19], the demand deposit contract optimally combines the two types of deposits that banks usually hold, a time deposit and a more typical demand deposit contract.

That is, at T=0 or ex-ante period, individuals deposit their unit of endowment at the bank and are offered a menu of contracts. At T=1 or interim period, depositors select their preferred contract (given their liquidity needs or information received).

A combination of these contracts is also possible, being the case of depositors allowed to withdraw at T=1 part of the second period withdrawal stream (subject to some early withdrawal penalty)[5].

The optimal contract choice for a deposit contract in the absence of interim information can be obtained by maximizing the aggregate expected utility of agents, that is, as the solution to the following problem:

$$\max_{c_{ij},K} \left\{ \underset{\tilde{R},\tilde{t}}{E} \left[\tilde{t}\, U^1(c_{11},\tilde{c}_{21},\rho_1) + (1-\tilde{t})\, U^2(c_{12},\tilde{c}_{22},\rho_2) \right] \right\} \qquad [7]$$

$$\text{s.t.} \quad \begin{aligned} &\tilde{t}c_{11} + (1-\tilde{t})c_{12} \leq K \\ &\tilde{t}\tilde{c}_{21} + (1-\tilde{t})\tilde{c}_{22} \leq (1-K)\tilde{R} \end{aligned} \qquad [8]$$

$$\underset{\tilde{R}}{E} \left[U^j(c_{1j},\tilde{c}_{2j},\rho_j) \right] \geq \underset{\tilde{R}}{E} \left[U^j(c_{1i},\tilde{c}_{2i},\rho_j) \right] \text{ for } i \neq j;\ i,j=1,2 \qquad [9]$$

where:

c_{1j} Consumption at time T=1 for the type j agent

$\tilde{c}_{2j}$ Consumption at time T=2 for the type j agent dependent on the random return $\tilde{R}$ ($c_{2j}(\tilde{R})$)

K Investment in the liquid short-lived asset at T=0

$1-K$ Investment in the illiquid long-lived asset at T=0

$\tilde{R}$ Random return of the long-lived asset at T=2.

The first two constraints represent resource balance constraints while the last two show the incentive compatibility constraints that guarantee that type-1 depositors will prefer their withdrawal stream to the type-2 withdrawal stream and viceversa, that is, the contract is designed so that depositors self-select their type contract.

Given that the ex-ante probability of *Rl* is sufficiently small, the ex-ante contract has been approximated ignoring consumption changes produced by interim signals. Moreover, the bank solves the ex-ante contract for the average t [6]. These simplifications are instrumental for finding an analytical solution to the ex-ante contract.

Applying Kunh-Tucker conditions, the optimal solution is obtained.

Lemma 1.- The optimal demand deposit contract satisfies: $c_{11}>c_{12}$ and $c_{22}>c_{21}\approx 0$

Proof: See Appendix A.1 for a detailed resolution of the problem.

2.2.- *The ex-post problem*

In the interim stage, the liquidity and information shocks are realized, and so every individual learns his or her type and also some type-2 agents will be informed about the return of the long-term asset at T=2. Given the liquidity needs or information received by individuals, they will select a contract to maximize their utility function. The choices of the individuals are defined by the dimensionless coefficients μ_1, μ_I and μ_2 respectively, explained in footnote 5.

The behaviour of the different agents is formulated as follows:

(i) Type-1 agents

The value of μ_1 is chosen in order to maximize their utility function and subject to their two period constraint; that is:

$$\begin{aligned} \max_{\mu_1} \quad & U^1(c_1,c_2,\rho_1)=\max_{\mu_1}\left\{\rho_1\frac{(k+c_1)^{1-\gamma}}{1-\gamma}+(1-\rho_1)E\left[\frac{(k+c_2)^{1-\gamma}}{1-\gamma}\right]\right\} \\ \text{s.t} \quad & c_1=c_{12}+\mu_1(c_{11}-c_{12}) \\ & c_2=c_{21}+(1-\mu_1)(c_{22}-c_{21}) \\ & \mu_1\leq 1 \end{aligned} \qquad [10]$$

The solution to the type-1 problem is given by Lemma 2:

Lemma 2.- Absent any information, type-1 agents will always select their own contract, that is, the optimal solution to the type-1 problem is $\mu_1^*=1$ $(c_{11},\ c_{21})$.

Proof: See Appendix A.2.

The solution to the type-1 problem is trivial, as these individuals do not care about second period consumption, therefore they will withdraw to consume in the interim period.

(ii) Informed type-2 agents.

In each state and conditional on the information about $\tilde{R}$ they solve the following problem:

$$\max_{\mu_I} U^I(c_1,c_2,\rho_2) = \max_{\mu_I}\left\{\rho_2\frac{(k+c_1)^{1-\gamma}}{1-\gamma}+(1-\rho_2)E\left[\frac{(k+c_2)^{1-\gamma}}{1-\gamma}\Big/\tilde{R}\right]\right\} \qquad [11]$$

$$\text{with:}\quad \begin{aligned} &c_1=c_{12}+\mu_I(c_{11}-c_{12})\\ &c_2=c_{21}+(1-\mu_I)(c_{22}-c_{21})\\ &\mu_I\leq 1\end{aligned} \qquad [12]$$

There are two different values for μ_I, depending on the information about $\tilde{R}$ received by these agents at $T=1$.

These solutions are defined by the following lemmas:

Lemma 3.- If type-2 agents receive positive information concerning the asset's return, they would choose a combination of the two contracts, that is:

$$0 \nless \mu_I=\frac{B_{2I}(k+c_{22})-(k+c_{12})}{B_{2I}(c_{22}-c_{21})+(c_{11}-c_{12})}<1 \quad \text{with} \quad B_{2I}=\left(\frac{c_{22}-c_{21}}{c_{11}-c_{12}}\,\frac{1-\rho_2}{\rho_2}\right)^{-\frac{1}{\gamma}} \qquad [13]$$

Proof: See Appendix A.2.

Lemma 4.- If type-2 agents receive negative information concerning the asset's return they will always claim the type-1 contract that is, the optimal solution is $\mu_I^*=1$ $(c_{11},\, c_{21})$.

Proof: See Appendix A.2.

(iii) Uninformed type-2 agents.

These agents maximize expected utility conditional on the observation of a noisy signal, which is the withdrawal queue size at the bank, or equivalently, the level of aggregate demanded consumption (CT) at $T=1$. The coefficient $\mu_2=\mu_2(CT)$ is chosen in order to maximize:

$$\max_{\mu_2} U^2(c_1,c_2,\rho_2) = \max_{\mu_2}\left\{\frac{(k+c_1)^{1-\gamma}}{1-\gamma} + E\left[\frac{(k+\tilde{c}_2)^{1-\gamma}}{1-\gamma} \Big/ CT\right]\right\}$$

$$\text{s.t.} \quad \begin{aligned} &c_1 = c_{12} + \mu_2(c_{11}-c_{12}) \\ &c_2 = c_{21} + (1-\mu_2)(c_{22}-c_{21}) \\ &\mu_2 \leq 1 \end{aligned} \tag{14}$$

The solution to the uninformed agents' problem is given by:

$$0<\mu_2=\frac{B_{CT}(k+c_{22})-(k+c_{12})}{B_{CT}(c_{22}-c_{21})+(c_{11}-c_{12})}\leq 1 \text{ with } B_{CT}=\left(\frac{c_{22}-c_{21}}{c_{11}-c_{12}}\,\frac{1-\rho_2}{\rho_2}\,\pi_1^*\right)^{-\frac{1}{\gamma}} \tag{15}$$

and where π_1^* represents the sum of probabilities of all the states of nature that give for the same value of μ_2 the same value of aggregate consumption, and for which $\tilde{R}=R_h$.

Proof: See Appendix A.2.

3.- Condition to assure a panic run

Bank runs occur whenever uninformed type-2 agents start making type-1 withdrawals upon observation of aggregate consumption at $T=1$. Conditions for both information-induced and pure panic runs to occur are given by Proposition 1.

Proposition 1.- In the model, bank runs occur as a unique equilibrium, if the following conditions hold:

$$\begin{aligned} &\rho_2\frac{(k+c_{11})^{1-\gamma}}{1-\gamma}+(1-\rho_2)\left[\frac{(k+c_{21})^{1-\gamma}}{1-\gamma}\pi_{1_{\mu=1}}+\frac{\left(k+c_{21}\frac{Rl}{Rh}\right)^{1-\gamma}}{1-\gamma}\pi_{2_{\mu=1}}\right]> \\ &>\rho_2\frac{(k+c_{12})^{1-\gamma}}{1-\gamma}+(1-\rho_2)\left[\frac{(k+c_{22})^{1-\gamma}}{1-\gamma}\pi_{1_{\mu=0}}+\frac{\left(k+c_{22}\frac{Rl}{Rh}\right)^{1-\gamma}}{1-\gamma}\pi_{2_{\mu=0}}\right] \end{aligned} \tag{16}$$

$$\rho_2(k+c_1)^{-\gamma}(c_{11}-c_{12})-(1-\rho_2)(k+c_2)^{-\gamma}\pi_{2_{\mu=1}}>0 \tag{17}$$

where:

$$\pi_{1_{\mu=1}} = \frac{(1-p)(1-q)}{1-q+pq} \qquad \pi_{2_{\mu=1}} = \frac{p}{1-q+pq}$$
$$\pi_{1_{\mu=0}} = \frac{(1-p)r_2}{r_1pq+r_2(1-q)+r_2(1-p)q} \qquad \pi_{2_{\mu=0}} = \frac{p[r_1q+r_2(1-q)]}{r_1pq+r_2(1-q)+r_2(1-p)q} \tag{18}$$

and c_1, c_2 as defined in the uninformed type-2 agents maximization problem.

$\pi_{1_{\mu=1}}$ and $\pi_{2_{\mu=1}}$ represent the sum of probabilities of all the states of nature that give for $\mu=1$ the same value of aggregate consumption ($\pi_{1_{\mu=1}}$ if $\tilde{R}=R_h$ and $\pi_{2_{\mu=1}}$ if $\tilde{R}=R_l$). Equivalently, $\pi_{1_{\mu=0}}$ and $\pi_{2_{\mu=0}}$ represent the sum of probabilities of all the states of nature that give for $\mu=0$ the same value of aggregate consumption ($\pi_{1_{\mu=0}}$ if $\tilde{R}=R_h$ and $\pi_{2_{\mu=0}}$ if $\tilde{R}=R_l$).

Proof: See Appendix B.

If these conditions are satisfied there are bank-runs in states 3, 4 and 6 and the levels of aggregate demanded consumption are the ones given by column 5 in Table I. It is observed that aggregate demanded consumption in those states is c_{11}, which exceeds ex-ante planned consumption at T=1 or the investment in the liquid asset K. In fact,

$$c_{11} > t\,c_{11} + (1-t)c_{12} \quad \Rightarrow \quad (1-t)c_{11} > (1-t)c_{12} \tag{19}$$

and $c_{11} > c_{12}$ by definition of the optimal deposit contract.

Whenever the withdrawal queue size at the bank exceeds the ex-ante investment in the liquid asset, the bank suspends convertibility, as will be seen in subsection 4.1.

In all these cases, bank runs occur as a unique equilibrium. In states 3 and 6 there exist information-induced runs as there is a negative information shock. However in state 4 there is a pure panic run as there is no adverse information held by any agent in this state.

4.- Public intervention

Coordination problems derived from deposit contracting are considered important due to the fear of systematic risk. An example may be found in the banking panics that ocurred in the 1930s and the strict regulation that was imposed as a reaction to the crisis. In spite of these measures being effective in preventing bank runs, they have created some additional problems, which provides some rationale for banking regulation. Deposit insurance exemplifies this situation as is considered the most effective tool to prevent runs, but with a high cost associated to it. Banks are not only induced to take excessive risks but depositors have no incentives to monitor their banks (moral hazard problem).

The aim of this section is to compare suspension of convertibility versus deposit insurance, given their relative benefits and costs (of randomization in meeting liquidity needs or deadweight taxation).

4.1.- Suspension of convertibility

It is a measure that banks have historically used against runs, in the pre-deposit insurance era.

In the model suspension of convertibility occurs at the level of the highest proportion of early diers, that is:

$$K + G = t_2 c_{11} + (1 - t_2) c_{12} \qquad [20]$$

where K is the ex-ante investment planned by the bank in the liquid asset and G is a subsidy received from the government in order to cover withdrawals up to the highest proportion of early diers (acting like a lender-of-last-resort).

However, in the ex-post situation, there may be some states of nature for which the value of aggregate consumption $\tilde{CT}_1$ is greater than the assumed ex-ante one.

The value of the aggregate demanded consumption $\tilde{CT}_1$ in state $\tilde{\theta}=(\tilde{t},\ \tilde{\alpha},\ \tilde{R})$ at $T=1$ is given by the expression:

$$\tilde{CT}_1 = \tilde{t} cc_{11} + \tilde{\alpha}(1-\tilde{t}) c\tilde{c}_{12I} + (1-\alpha)(1-\tilde{t}) cc_{12} \qquad [21]$$

and where cc_{11} represents the consumption at date 1 of type-1 agents (value of c_1 obtained in (i)) and cc_{12I}, cc_{12} that of informed and uninformed type-2 agents respectively (obtained in (ii) and (iii)).

Then, the deficit $\Delta \tilde{CT}_1$ is defined as follows:

$$\begin{aligned} \Delta \tilde{CT}_1 &= \tilde{CT}_1 - (K+G) \quad && \text{if} \quad \tilde{CT}_1 \geq K+G \\ \Delta \tilde{CT}_1 &= 0 && \text{if} \quad \tilde{CT}_1 < K+G \end{aligned} \qquad [22]$$

This deficit in consumption has to be shared among the different agents. In the model, it is assumed that each depositor arrives in line randomly at the bank and the depositors are then treated on a first-come-first-served basis. Let $\tilde{\beta}$ be the random proportion of agents of each type that supports the deficit with respect to the total number of the same type of agents, thus, these agents will receive what was planned in the ex-ante analysis, i.e.:

$$\tilde{\beta} = \frac{\Delta \tilde{CT}_1}{(1-\tilde{t})\left[\tilde{\alpha}\,(cc_{12I} - c_{12}) + (1-\tilde{\alpha})\,(cc_{12} - c_{12})\right] + \tilde{t}\,(cc_{11} - c_{12})} \qquad [23]$$

It should be noted that $\tilde{\beta} \geq 0$ because μ_1, μ_I and μ_2 are positive numbers and cc_{11}, cc_{12I} and $cc_{12} > c_{12}$. The consumptions are then:

Type-1 agents:

$$\begin{aligned} &cc_{11}^{s_1} = cc_{11} && \text{for the first } (1-\tilde{\beta})\,\tilde{t} \\ &cc_{21}^{s_1} = cc_{21} && \text{type-1 agents} \\ &cc_{11}^{s_2} = c_{12} && \text{for the last } \tilde{\beta}\,\tilde{t} \\ &cc_{21}^{s_2} = cc_{21} && \text{type-1 agents} \end{aligned} \qquad [24]$$

Informed type-2 agents:

$$\begin{aligned} &cc_{12I}^{s_1} = cc_{12I} && \text{for the first } (1-\tilde{\beta})\,\tilde{\alpha}\,(1-\tilde{t}) \text{ informed} \\ &cc_{22I}^{s_1} = cc_{22I} && \text{type-2 agents} \\ &cc_{12I}^{s_2} = c_{12} && \text{for the last } \tilde{\beta}\,\tilde{\alpha}\,(1-\tilde{t}) \text{ informed} \\ &cc_{22I}^{s_2} = cc_{22I} && \text{type-2 agents} \end{aligned} \qquad [25]$$

Uninformed type-2 agents:

$$\begin{aligned} &cc_{12}^{s_1} = cc_{12} && \text{for the first } (1-\tilde{\beta})(1-\tilde{\alpha})(1-\tilde{t}) \text{ uninformed} \\ &cc_{22}^{s_1} = cc_{22} && \text{type-2 agents} \\ &cc_{12}^{s_2} = c_{12} && \text{for the last } \tilde{\beta}\,(1-\tilde{\alpha})(1-\tilde{t}) \text{ uninformed} \\ &cc_{22}^{s_2} = cc_{22} && \text{type-2 agents} \end{aligned} \qquad [26]$$

Given these modified consumption levels after suspension, the aggregate expected utility, U_T, is defined as follows:

$$
\begin{aligned}
EU_T = \sum_{\theta} U_T p(\theta) = \sum_{\theta} \Bigg\{ & \left[\rho_1 \frac{\left(k + cc_{11}^{s_1}\right)^{1-\gamma}}{1-\gamma} + (1-\rho_1) \frac{\left(k + \frac{R^\theta}{Rh} cc_{21}^{s_1}\right)^{1-\gamma}}{1-\gamma} \right] t^\theta (1-\beta^\theta) + \\
& \left[\rho_1 \frac{\left(k + cc_{11}^{s_2}\right)^{1-\gamma}}{1-\gamma} + (1-\rho_1) \frac{\left(k + \frac{R^\theta}{Rh} cc_{21}^{s_2}\right)^{1-\gamma}}{1-\gamma} \right] t^\theta \beta^\theta + \\
& \left[\rho_2 \frac{\left(k + cc_{12I}^{s_1}\right)^{1-\gamma}}{1-\gamma} + (1-\rho_2) \frac{\left(k + \frac{R^\theta}{Rh} cc_{22I}^{s_1}\right)^{1-\gamma}}{1-\gamma} \right] (1-t^\theta)\, \alpha^\theta (1-\beta^\theta) + \\
& \left[\rho_2 \frac{\left(k + cc_{12I}^{s_2}\right)^{1-\gamma}}{1-\gamma} + (1-\rho_2) \frac{\left(k + \frac{R^\theta}{Rh} cc_{22I}^{s_2}\right)^{1-\gamma}}{1-\gamma} \right] (1-t^\theta)\, \alpha^\theta \beta^\theta + \\
& \left[\rho_2 \frac{\left(k + cc_{12}^{s_1}\right)^{1-\gamma}}{1-\gamma} + (1-\rho_2) \frac{\left(k + \frac{R^\theta}{Rh} cc_{22}^{s_1}\right)^{1-\gamma}}{1-\gamma} \right] (1-t^\theta)(1-\alpha^\theta)(1-\beta^\theta) + \\
& \left[\rho_2 \frac{\left(k + cc_{12}^{s_2}\right)^{1-\gamma}}{1-\gamma} + (1-\rho_2) \frac{\left(k + \frac{R^\theta}{Rh} cc_{22}^{s_2}\right)^{1-\gamma}}{1-\gamma} \right] (1-t^\theta)(1-\alpha^\theta)\beta^\theta \Bigg\} p(\theta)
\end{aligned}
\qquad [27]
$$

where $\theta = 1,..,6$

The welfare measure to be considered in this study will be:

$$
\text{cert.equivalent}(EU_T) - \left[\sum_{\theta} Gp(\theta)\right](1+s) \qquad \theta = 3,\ 4,\ 5,\ 6 \qquad [28]
$$

The subsidy G is received from the government in order to cover withdrawals up to the highest proportion of early diers. It would be like a deadweight tax on individuals of value s.

4.2.- Deposit insurance

In the model it is assumed that there is government deposit insurance of the bank deposits at T=2. This insurance removes the incentives of agents to become informed and so information-induced runs will no longer occur. In this case, it is assumed that agents will consume what was planned in the ex-ante contract.

Whenever the realized benefit at T=2, $\tilde{R}\ (1-K)$ is less than the assumed consumption for that period, that is, $tc_{21} + (1-t)c_{22}$, the difference is always supplied by the government. The expected cost of the insurance would be:

$$Cd=\sum_{\theta}\left[tc_{21}+(1-t)c_{22}-R^{\theta}(1-K)\right]p(\theta) \qquad [29]$$

The expected cost of the insurance may be considered also as a deadweight tax on individuals and so the welfare measure for deposit insurance will be:

$$\text{cert. equivalent}(U\text{exante})-Cd(1+s)-\left[\sum_{\theta}G\,p(\theta)\right](1+s) \qquad \theta=4,\ 5,\ 6 \qquad [30]$$

where G is also the subsidy received from the government to cover the liquidity shock, that is, in those states of nature in which t_2 is realized.

5.- Comparison among the different public intervention measures: numerical examples

In the numerical simulations, it has been assumed the lowest proportion of type-1 agents (t_1) is realized with a high probability $(r_1=0.99)$. The motivation for this assumption is to create confusion between a large withdrawal queue size at the bank, due to a high liquidity shock (t_2 realized), or a negative information shock. It is also assumed that the probability of the low value of the random return occuring (Rl) is sufficiently small $(p=0.10)$, and this in turn simplifies the ex-ante contract maximization problem. The data used in the simulations are shown in Table II.

The sensitivity analysis has been carried out with respect to the relative risk-aversion coefficient (γ), and the standard deviation of the random return, $\tilde{R}$, (keeping the mean return constant and increasing the dispersion of $\tilde{R}$)[7].

Figure 1 shows the certainty equivalent of the utility attained with suspension of convertibility minus deposit insurance, as a function of the relative risk aversion coefficient (γ) and for different values of the standard deviation of the random return (measured as $\sigma/\overline{R}$). It can be seen that for low values of γ, suspension would be welfare superior. On the other hand, deposit insurance would be better for intermediate values of γ (up to $\gamma=1.70$), lastly, from then on, suspension yields again higher utility[8]. As σ increases, suspension is welfare superior for nearly all levels of risk-aversion and for *very risky* investments *suspension would be always the preferred measure.*

Figure 2 gives the certainty equivalent of the utility attained with suspension of convertibility minus deposit insurance as a function of the standard deviation of the random return (measured as $\sigma/\bar{R}$) and for different values of γ. It can be observed that as $\sigma/\bar{R}$ increases suspension becomes the preferred measure to cope with runs.

These figures suggest that deposit insurance should only be restricted to finance low risk assets, that is, it gives support for the "narrow banking" proposal that has been currently discussed in the literature.

With respect to the other parameters, variations in the value of deadweight tax (s) do not have a significant effect on the value of the welfare measures and the sensitivity analysis with respect to the proportion of early diers is ambiguous as it means also a variation in the proportion of informed agents (given the parameter restrictions defined in equation [5]). Finally, one possible extension would be to let both agents have interior preferences, with type-1 individuals deriving relatively more utility from consumption in the first period with respect to type-2. In this case, however, only numerical solutions may be possible.

6.- Conclusions and suggestions for further research

This paper introduces risk-averse preferences in Chari and Jagannathan's model. A first motivation for this extension was to give a positive role for a financial intermediary in the economy. The introduction of risk-aversion in Chari and Jagannathan's model, implies an ex-ante definition of the optimal insurance contract. This transformation service (through the demand deposit contract) is one of the important functions performed by banks.

Once the banking contract or ex-ante program has been designed, all agents solve their maximization problem in the interim period conditional on their information (if any) and they decide on their level of consumption for both periods. Conditions to assure bank-runs (both information-induced and "panic" runs) are derived.

A second motivation for this extension was to complete Chari and Jagannathan's welfare analysis, by comparing suspension of convertibility versus deposit insurance, given their relative benefits and costs (of randomization in meeting liquidity needs or deadweight taxation).

The numerical results have shown that in all cases, as the dispersion of the random return increases, suspension of convertibility improves with respect to deposit insurance. For very risky investments, suspension would be the preferred measure to prevent runs.

In general, the results of this paper support a radical policy proposal that aims to restrict insured liabilities to finance very low risk assets. This proposal, known as "narrow banking", would consist in dividing the banking industry into two types of banks: a "narrow" group of banks, whose deposits would be insured, and who are restricted in their assets' choices and a broad class of banks, with greater flexibility in the use of their uninsured deposits. The narrow banking idea has received support in the literature, among others, by Kareken [21], Boot and Greenbaum [7], Jacklin [20] and Craine [10].

Appendix

A.- Maximization problems

A.1.- Ex-ante Contract

In the ex ante contract maximization problem both $\tilde{t}$ (the proportion of type-1 agents) and $\tilde{R}$ (the return on the long-term asset) are random variables.

A first simplification to the problem has been done by substituting $\tilde{t}$ by its expected value $t = t_1 r_1 + t_2 r_2$. The bank solves its ex-ante program for the average proportion of type-1 agents[9].

Taking this into consideration, the maximization problem defined by equations [7], [8] and [9] is approximated as follows:

$$\max_{c_{ij}} \left\{ \underset{\tilde{R}}{E} \left[t\, U^1(c_{11}, \tilde{c}_{21}, \rho_1) + (1-t)\, U^2(c_{12}, \tilde{c}_{22}, \rho_2) \right] \right\} \quad [31]$$

$$\text{s.t.} \quad t\left(c_{11} + \frac{\tilde{c}_{21}}{\tilde{R}}\right) + (1-t)\left(c_{12} + \frac{\tilde{c}_{22}}{\tilde{R}}\right) = 1 \quad [32]$$

$$\underset{\tilde{R}}{E} \left[U^j(c_{1j}, \tilde{c}_{2j}, \rho_j) \right] \geq \underset{\tilde{R}}{E} \left[U^j(c_{1i}, \tilde{c}_{2i}, \rho_j) \right] \text{ for } i \neq j;\ i,j = 1,2 \quad [33]$$

where the two resource constraints have been substituted by a unique constraint.

The second type of uncertainty reflects the fact that, having invested in a risky technology, the bank may not be able to make its promised second period payments in full. One way to think of this, is that the bank promises an amount (c_{21}, c_{22}) it will be able to pay if $R = Rh$. If $R = Rl$ really occurs, the bank is considered insolvent and depositors get $\frac{Rl}{Rh}$ of their promised payments. It is then assumed that:

$$\tilde{c}_{2j} = c_{2j} \frac{\tilde{R}}{Rh} \quad (j = 1,2) \quad [34]$$

Given this dependence between consumption and returns, the above maximization problem is reformulated as follows[10]:

$$\max_{c_{ij}} \left\{ \begin{array}{l} t\left(\rho_1 \dfrac{(k+c_{11})^{1-\gamma}}{1-\gamma} + (1-\rho_1) E\left[\dfrac{(k+c_{21})^{1-\gamma}}{1-\gamma}\right]\right) + \\ +(1-t)\left(\rho_2 \dfrac{(k+c_{12})^{1-\gamma}}{1-\gamma} + (1-\rho_2) E\left[\dfrac{(k+c_{22})^{1-\gamma}}{1-\gamma}\right]\right) \end{array} \right\} \qquad [35]$$

s.t.

$$\begin{array}{l} t\left(c_{11} + \dfrac{c_{21}}{Rh}\right) + (1-t)\left(c_{12} + \dfrac{c_{22}}{Rh}\right) = 1 \\ \rho_1 \dfrac{(k+c_{11})^{1-\gamma}}{1-\gamma} + (1-\rho_1) E\left[\dfrac{(k+c_{21})^{1-\gamma}}{1-\gamma}\right] \geq \rho_1 \dfrac{(k+c_{12})^{1-\gamma}}{1-\gamma} + (1-\rho_1) E\left[\dfrac{(k+c_{22})^{1-\gamma}}{1-\gamma}\right] \\ \rho_2 \dfrac{(k+c_{12})^{1-\gamma}}{1-\gamma} + (1-\rho_2) E\left[\dfrac{(k+c_{22})^{1-\gamma}}{1-\gamma}\right] \geq \rho_2 \dfrac{(k+c_{11})^{1-\gamma}}{1-\gamma} + (1-\rho_2) E\left[\dfrac{(k+c_{21})^{1-\gamma}}{1-\gamma}\right] \end{array} \qquad [36]$$

and where λ_1, λ_2, λ_3 are the multipliers associated with the corresponding resource and incentive constraints.

In this maximization problem, it can be shown that the incentive constraint for type-1 agents is never binding and that of type-2 agents *may be* binding (depending on the exogenous parameters that are considered, as will be seen below).

The F.O.C to the above maximization problem are:

$$\begin{array}{ll} \dfrac{\partial L}{\partial c_{11}} = (k+c_{11})^{-\gamma}(t\rho_1 - \rho_2\lambda_3) - t\lambda_1 = 0 & [a] \\ \dfrac{\partial L}{\partial c_{21}} = \left[(k+c_{21})^{-\gamma}(1-p) + p\left(k + c_{21}\dfrac{Rl}{Rh}\right)^{-\gamma}\dfrac{Rl}{Rh}\right]\left[t(1-\rho_1) - (1-\rho_2)\lambda_3\right] - \dfrac{t}{Rh}\lambda_1 = 0 & [b] \\ \dfrac{\partial L}{\partial c_{12}} = (k+c_{12})^{-\gamma}\left[(1-t)\rho_2 + \rho_2\lambda_3\right] - (1-t)\lambda_1 = 0 & [c] \\ \dfrac{\partial L}{\partial c_{22}} = \left[(k+c_{22})^{-\gamma}(1-p) + p\left(k + c_{22}\dfrac{Rl}{Rh}\right)^{-\gamma}\dfrac{Rl}{Rh}\right]\left[(1-t)(1-\rho_2) + (1-\rho_2)\lambda_3\right] - \dfrac{(1-t)}{Rh}\lambda_1 = 0 & [d] \\ \dfrac{\partial L}{\partial \lambda_1} = 1 - t\left(c_{11} + \dfrac{c_{21}}{Rh}\right) - (1-t)\left(c_{12} + \dfrac{c_{22}}{Rh}\right) = 0 & [e] \\ \dfrac{\partial L}{\partial \lambda_3} = \rho_2 \dfrac{(k+c_{12})^{1-\gamma}}{1-\gamma} + (1-\rho_2) E\dfrac{(k+c_{22})^{1-\gamma}}{1-\gamma} - \rho_2 \dfrac{(k+c_{11})^{1-\gamma}}{1-\gamma} - (1-\rho_2) E\dfrac{(k+c_{21})^{1-\gamma}}{1-\gamma} = 0 & [f] \end{array} \qquad [37]$$

The new unknowns are introduced:

$$\begin{array}{l} \hat{c}_{1j} = c_{1j} + k \\ \hat{c}_{2j} = c_{2j} + k \qquad \text{where: } j = 1,\ 2 \end{array} \qquad [38]$$

Given that $\rho_1 \to 1$, it is assumed the corner solution $\hat{c}_{21} = k$ (or $c_{21} = 0$).

(i) The two incentive constraints are never binding ($\lambda_2 = 0,\ \lambda_3 = 0$).

The first order conditions become:

$$\frac{\partial L}{\partial \hat{c}_{11}} = \hat{c}_{11}^{-\gamma} t \rho_1 - t\lambda_1 = 0 \qquad [a]$$

$$\frac{\partial L}{\partial \hat{c}_{21}} = \hat{c}_{21}^{-\gamma} (1-p) t (1-\rho_1) - \frac{t}{Rh}\lambda_1 \leq 0 \qquad [b]$$

$$\frac{\partial L}{\partial \hat{c}_{12}} = \hat{c}_{12}^{-\gamma} (1-t)\rho_2 - (1-t)\lambda_1 = 0 \qquad [c] \qquad [39]$$

$$\frac{\partial L}{\partial \hat{c}_{22}} = \hat{c}_{22}^{-\gamma} (1-p)(1-t)(1-\rho_2) - \frac{(1-t)}{Rh}\lambda_1 = 0 \qquad [d]$$

$$\frac{\partial L}{\partial \lambda_1} = 1 + \left(1 + \frac{1}{Rh}\right)k - t\left(\hat{c}_{11} + \frac{\hat{c}_{21}}{Rh}\right) - (1-t)\left(\hat{c}_{12} + \frac{\hat{c}_{22}}{Rh}\right) = 0 \qquad [e]$$

From [39][a] and [39][c]:

$$\hat{c}_{12} = \left(\frac{\rho_2}{\rho_1}\right)^{\frac{1}{\gamma}} \hat{c}_{11} \qquad [40]$$

From [39][a] and [39][d]:

$$\hat{c}_{22} = \left[\frac{(1-p)(1-\rho_2)Rh}{\rho_1}\right]^{\frac{1}{\gamma}} \hat{c}_{11} \qquad [41]$$

Substituting [40] and [41] in [39][e] the value of $\hat{c}_{11}$ is obtained.

$$\hat{c}_{11} = \frac{1 + k + (1-t)\dfrac{k}{Rh}}{t + (1-t)\left\{\left(\dfrac{\rho_2}{\rho_1}\right)^{\frac{1}{\gamma}} + Rh^{\frac{1-\gamma}{\gamma}}\left[\dfrac{(1-\rho_2)(1-p)}{\rho_1}\right]^{\frac{1}{\gamma}}\right\}} \qquad [42]$$

In this case it is assumed that the incentive constraint for type-2 agents is not binding, that is:

$$\rho_2 \frac{\hat{c}_{12}^{1-\gamma}}{1-\gamma} + (1-\rho_2) E\frac{\hat{c}_{22}^{1-\gamma}}{1-\gamma} - \rho_2 \frac{\hat{c}_{11}^{1-\gamma}}{1-\gamma} - (1-\rho_2) E\frac{\hat{c}_{21}^{1-\gamma}}{1-\gamma} \geq 0 \qquad [43]$$

Substituting the optimal consumption levels in the above expression, a condition on the relative risk-aversion coefficient (γ) for this case to hold is obtained.

(ii) The incentive constraint for type-1 agents is not binding and that of type-2 is binding ($\lambda_2 = 0,\ \lambda_3 > 0$).

The first order conditions become:

$$\begin{aligned}
&\frac{\partial L}{\partial \hat{c}_{11}} = \hat{c}_{11}^{-\gamma}(t\rho_1 - \rho_2\lambda_3) - t\lambda_1 = 0 && [a]\\
&\frac{\partial L}{\partial \hat{c}_{21}} = \hat{c}_{21}^{-\gamma}(1-p)\left[t(1-\rho_1) - (1-\rho_2)\lambda_3\right] - \frac{t}{Rh}\lambda_1 \leq 0 && [b]\\
&\frac{\partial L}{\partial \hat{c}_{12}} = \hat{c}_{12}^{-\gamma}\left[(1-t)\rho_2 + \rho_2\lambda_3\right] - (1-t)\lambda_1 = 0 && [c]\\
&\frac{\partial L}{\partial \hat{c}_{22}} = \hat{c}_{22}^{-\gamma}(1-p)\left[(1-t)(1-\rho_2) + (1-\rho_2)\lambda_3\right] - \frac{(1-t)}{Rh}\lambda_1 = 0 && [d]\\
&\frac{\partial L}{\partial \lambda_1} = 1 + \left(1 + \frac{1}{Rh}\right)k - t\left(\hat{c}_{11} + \frac{\hat{c}_{21}}{Rh}\right) - (1-t)\left(\hat{c}_{12} + \frac{\hat{c}_{22}}{Rh}\right) = 0 && [e]\\
&\frac{\partial L}{\partial \lambda_3} = \rho_2\frac{\hat{c}_{12}^{1-\gamma}}{1-\gamma} + (1-\rho_2)E\frac{\hat{c}_{22}^{1-\gamma}}{1-\gamma} - \rho_2\frac{\hat{c}_{11}^{1-\gamma}}{1-\gamma} - (1-\rho_2)E\frac{\hat{c}_{21}^{1-\gamma}}{1-\gamma} = 0 && [f]
\end{aligned} \qquad [44]$$

From [44][c]:

$$\lambda_3 = -(1-t)\left[1 - \frac{\lambda_1}{\hat{c}_{12}^{-\gamma}\rho_2}\right] \qquad [45]$$

From [44][d]:

$$\lambda_3 = -(1-t)\left[1 - \frac{\lambda_1}{\hat{c}_{22}^{-\gamma}Rh(1-p)(1-\rho_2)}\right] \qquad [46]$$

From [45] and [46]:

$$\hat{c}_{22} = \left[(1-p)Rh\frac{1-\rho_2}{\rho_2}\right]^{\frac{1}{\gamma}}\hat{c}_{12} \qquad [47]$$

From [44][a]:

$$\lambda_3 = t\left[\frac{\rho_1}{\rho_2} - \frac{\lambda_1}{\hat{c}_{11}^{-\gamma}\rho_2}\right] \qquad [48]$$

From [45] and [48]:

$$\hat{c}_{11}^{-\gamma} = \frac{D B \rho_2 \hat{c}_{12}^{-\gamma}}{\hat{c}_{12}^{-\gamma}(B\rho_1 - 1) + D} \qquad [49]$$

where:

$$M = 1 + k + \frac{k-t}{Rh} + \frac{\hat{c}_{12}}{Rh}(t-1)\left\{\left[(1-p)Rh\frac{1-\rho_2}{\rho_2}\right]^{1/\gamma} + Rh\right\}$$
$$B = -\frac{t}{(1-t)\rho_2} \qquad D(\hat{c}_{12}) = \frac{\lambda_1}{\rho_2} \qquad [50]$$

Substituting in [44][e], [47] and [49]:

$$\lambda_1 = D\rho_2 = \frac{\left[\frac{M}{t}\right]^{-\gamma}\hat{c}_{12}^{-\gamma}(B\rho_1 - 1)}{B\rho_2\hat{c}_{12}^{-\gamma} - \left[\frac{M}{t}\right]^{-\gamma}}\rho_2 \qquad [51]$$

Subsituting in [44][f]:

$$(1-\rho_2)(1-p)\left\{\left[(1-p)Rh\frac{1-\rho_2}{\rho_2}\right]^{\frac{1-\gamma}{\gamma}}\hat{c}_{12}^{1-\gamma} - k^{1-\gamma}\right\} - \rho_2\left\{\left[\frac{DB\rho_2\hat{c}_{12}^{-\gamma}}{\hat{c}_{12}^{-\gamma}(B\rho_1 - 1) + D}\right]^{\frac{\gamma-1}{\gamma}} - \hat{c}_{12}^{1-\gamma}\right\} = 0 \qquad [52]$$

The solution to the non linear equation [47] yields a value for $\hat{c}_{12}$, and from it the rest of the unknowns of the problem are obtained.

$$\hat{c}_{11} = \left[\frac{D(\hat{c}_{12})B\rho_2\hat{c}_{12}^{-\gamma}}{B\hat{c}_{12}^{-\gamma}\rho_1 - \hat{c}_{12}^{-\gamma} + D(\hat{c}_{12})}\right]^{-1/\gamma} \qquad \hat{c}_{21} = k \qquad \hat{c}_{22} = \left[\frac{1-\rho_2}{\rho_2}(1-p)Rh\right]^{\frac{1}{\gamma}}\hat{c}_{12} \qquad [53]$$

The ex-ante contract has been solved assuming $Rl \to 0$. The more general case in which $Rl \neq 0$ (sensitivity analysis with respect to $\tilde{R}$) has been solved applying the Newton-Raphson technique to the F.O.C in Equation [37].

A.2.- Ex-Post Problem

A.2.1.- Type-1 agents

The value of μ_1 is chosen in order to maximize their utility function and subject to their two period constraint; that is:

$$\max_{\mu_1} U^1(c_1,c_2) = \max_{\mu_1}\left\{\rho_1\frac{(k+c_1)^{1-\gamma}}{1-\gamma}+(1-\rho_1)\left[(1-p)\frac{(k+c_2)^{1-\gamma}}{1-\gamma}+p\frac{\left(k+c_2\dfrac{Rl}{Rh}\right)^{1-\gamma}}{1-\gamma}\right]\right\}$$

$$\text{s.t} \quad c_1=c_{12}+\mu_1(c_{11}-c_{12}) \qquad c_2=c_{21}+(1-\mu_1)(c_{22}-c_{21}) \qquad \mu_1\leq 1 \tag{54}$$

The F.O.C of the problem are:

$$\rho_1(k+c_1)^{-\gamma}(c_{11}-c_{12})-(1-\rho_1)(1-p)(k+c_2)^{-\gamma}(c_{22}-c_{21})-\lambda_1=0 \tag{55}$$

and given that $\rho_1 \to 1$ the multiplier associated with the constraint is:

$$\lambda_1=\rho_1(k+c_1)^{-\gamma}(c_{11}-c_{12})>0 \qquad \text{as:} \quad c_{11}>c_{12} \tag{56}$$

and therefore the constraint is always binding, that is, $\mu_1=1$

A.2.2.- Informed type-2 agents

In each state and conditional on the information about $\tilde{R}$ they solve the following problem:

$$\max_{\mu_I} = \rho_2\frac{(k+c_1)^{1-\gamma}}{1-\gamma}+(1-\rho_2)E\left[\frac{(k+c_2)^{1-\gamma}}{1-\gamma}\Big/\tilde{R}\right] \tag{57}$$

$$\text{with:} \quad c_1=c_{12}+\mu_I(c_{11}-c_{12}) \qquad c_2=c_{21}+(1-\mu_I)(c_{22}-c_{21}) \qquad \mu_I\leq 1 \tag{58}$$

There are two different values for μ_I, depending on the information about the random return ($\tilde{R}$) received by these agents at $T=1$.

(i) If $\tilde{R}=Rh$ is the information received at date 1, then the informed type-2 agents find their consumption by solving the following problem:

$$\max_{\mu_I} U^1(c_1,c_2) = \max_{\mu_I}\left\{\rho_2\frac{(k+c_1)^{1-\gamma}}{1-\gamma}+(1-\rho_2)\frac{(k+c_2)^{1-\gamma}}{1-\gamma}\right\} \tag{59}$$

$$\text{with} \quad \begin{aligned} &c_1 = c_{12} + \mu_I(c_{11} - c_{12}) \\ &c_2 = c_{21} + (1-\mu_I)(c_{22} - c_{21}) \\ &\mu_I \leq 1 \end{aligned} \tag{60}$$

The F.O.C of the problem are:

$$\rho_2 (k+c_1)^{-\gamma}(c_{11}-c_{12}) - (1-\rho_2)(k+c_2)^{-\gamma}(c_{22}-c_{21}) - \lambda_1 = 0 \tag{61}$$

with solution:

$$0 \nless \mu_2 = \frac{B_I(k+c_{22}) - (k+c_{12})}{B_I(c_{22}-c_{21}) + (c_{11}-c_{12})} < 1 \quad \text{with} \quad B_I = \left(\frac{c_{22}-c_{21}}{c_{11}-c_{12}} \, \frac{1-\rho_2}{\rho_2} \right)^{-\frac{1}{\gamma}} \tag{62}$$

(ii) If $\tilde{R}=Rl$ is the value of $\tilde{R}$ revealed to type-2 agents, then the level of consumption is obtained in a similar way as above:

$$\max_{\mu_I} U^1(c_1,c_2) = \max_{\mu_1} \left\{ \rho_2 \frac{(k+c_1)^{1-\gamma}}{1-\gamma} + (1-\rho_2) \frac{\left(k + c_2 \dfrac{Rl}{Rh}\right)^{1-\gamma}}{1-\gamma} \right\} \tag{63}$$

$$\text{with:} \quad \begin{aligned} &c_1 = c_{12} + \mu_I(c_{11} - c_{12}) \\ &\tilde{c}_2 = \tilde{c}_{21} + (1-\mu_I)(\tilde{c}_{22} - \tilde{c}_{21}) \\ &\mu_I \leq 1 \end{aligned} \tag{64}$$

The F.O.C of the problem are:

$$\rho_2 (k+c_1)^{-\gamma}(c_{11}-c_{12}) - (1-\rho_2)\left(k + c_2 \frac{Rl}{Rh}\right)^{-\gamma} \frac{Rl}{Rh}(c_{22}-c_{21}) - \lambda_1 = 0 \tag{65}$$

and therefore the value of λ_1 is:

$$\lambda_1 = \rho_2 (k+c_1)^{-\gamma}(c_{11}-c_{12}) > 0 \tag{66}$$

which implies $\mu_I = 1$

A.2.3.- Uninformed type-2 agents

Their maximization problem is given by:

$$\max_{\mu_2} U^2(c_1, c_2, \rho_2) = \max_{\mu_2}\left\{\frac{(k+c_1)^{1-\gamma}}{1-\gamma} + E\left[\frac{(k+\tilde{c}_2)^{1-\gamma}}{1-\gamma} \Big/ CT\right]\right\}$$

$$\text{s.t.} \quad c_1 = c_{12} + \mu_2(c_{11} - c_{12})$$
$$c_2 = c_{21} + (1-\mu_2)(c_{22} - c_{21})$$
$$\mu_2 \leq 1 \qquad [67]$$

The value of CT depends on each state of nature $\tilde{\theta} = (\tilde{t}, \tilde{\alpha}, \tilde{R})$ with probability $p(\tilde{\theta})$ and its expression is:

$$\tilde{CT} = \tilde{t}\, cc_{11} + (1-\tilde{t})[\tilde{\alpha}\, cc_{12I} + (1-\tilde{\alpha})cc_{12}] \qquad [68]$$

and where cc_{11} represents the consumption at date 1 of type-1 agents and cc_{12I}, cc_{12} that of informed and uninformed type-2 agents respectively.

The values of CT are shown in Table I. It should be observed that there for any $\mu_2(CT)$ or equivalently any consumption level (cc_{12}), uninformed agents may choose, there is always confusion between states 3 and 4 as type-1's consumption at T=1 (cc_{11}) is c_{11} and informed type-2's consumption in the case of a negative information shock (cc_{12I}) is also c_{11}.

More generally, let θ^*_{CT} be defined as the finite set $\theta^*_{1,CT}, \theta^*_{2,CT}, \ldots \theta^*_{N,CT}$ of all the states of nature that give for the same value of $\mu_2 = \mu_2(CT)$ the same value of aggregate consumption. The probability of state $\theta^*_{i,CT}$ is:

$$p(\theta^*_{i,CT}/CT) = \frac{p(\theta^*_{i,CT})}{\sum_{i=1}^{N} p(\theta^*_{i,CT})} \qquad [69]$$

The set of N states of nature can be divided into 2 groups. The first N_1 states correspond to $\tilde{R}=Rl$ and the last N_2 to the case $\tilde{R}=Rh$. On the other side, given the independence of the random variables equation [69] can be rewritten:

$$p(\theta^*_{i,CT}/ CT) = \frac{p(t^*_{i,CT})p(\alpha^*_{i,CT})}{\sum_{i=1}^{N} p(\theta^*_{i,CT})} p = \pi_i p \qquad \text{if} \quad 0<i<N_1 \qquad [70]$$

$$p(\theta^*_{i,CT}/CT)=\frac{p(t^*_{i,CT})p(\alpha^*_{i,CT})}{\sum_{i=1}^{N} p(\theta^*_{i,CT})}(1-p)=\pi_i(1-p) \qquad \text{if } \ N_1<i<N_2 \qquad [71]$$

And so the problem defined by [14], [67] is reformulated as follows:

$$\max_{\mu_2}\left\{\rho_2\frac{(k+c_1)^{1-\gamma}}{1-\gamma}+(1-\rho_2)\left[\frac{(k+c_2)^{1-\gamma}}{1-\gamma}\pi_1^*+\frac{\left(k+c_2\frac{Rl}{Rh}\right)^{1-\gamma}}{1-\gamma}\pi_2^*\right]\right\}$$
$$\begin{aligned} \text{s.t.} \quad & c_1=c_{12}+\mu_2(c_{11}-c_{12}) \\ & c_2=c_{21}+(1-\mu_2)(c_{22}-c_{21}) \\ & \mu_2\leq 1 \end{aligned} \qquad [72]$$

where:

$$\pi_1^*=(1-p)\sum_{i=1+N_1}^{N}\pi_i \qquad \pi_2^*=p\sum_{i=1}^{N_1}\pi_i \qquad [73]$$

The solution to the uninformed agents' problem is given by:

$$0<\mu_2=\frac{B_{CT}(k+c_{22})-(k+c_{12})}{B_{CT}(c_{22}-c_{21})+(c_{11}-c_{12})}\leq 1 \text{ with } B_{CT}=\left(\frac{c_{22}-c_{21}}{c_{11}-c_{12}}\frac{1-\rho_2}{\rho_2}\pi_1^*\right)^{-\frac{1}{\gamma}} \text{ and } \pi_1^*=\sum \text{ states nature } \tilde{R}=R_h \qquad [74]$$

B.- Proof of Proposition 1

Proposition 1 assures that the optimal choice of uninformed type-2 agents in states 3, 4 and 6 is $\mu_2^*=1$, that is, the choice of the type-1 contract.

First, considering states 3 and 4 and due to the existence of conditional probabilities, three different expressions for the utility function, $OF(\mu_2)$, can be written:

(i) If $\mu_2=1$ then aggregate consumption at date 1 would be the same for states 1, 3, 4 and 6 and so the conditional probabilities would be the ones given by $\pi_{1_{\mu=1}}$ and $\pi_{2_{\mu=1}}$ and the utility function is:

(ii) If $0<\mu_2<1$ then, there is only confusion between states 3 and 4 and the conditional probabilities would be given by:

$$OF_{(\mu_2=1)}=\rho_2\frac{(k+c_{11})^{1-\gamma}}{1-\gamma}+(1-\rho_2)\left[\frac{(k+c_{21})^{1-\gamma}}{1-\gamma}\pi_{1_{\mu=1}}+\frac{\left(k+c_{21}\frac{Rl}{Rh}\right)^{1-\gamma}}{1-\gamma}\pi_{2_{\mu=1}}\right] \quad [75]$$

$$\pi_{1_{\mu=1}}=\frac{(1-p)r_2(1-q)}{r_1pq+r_2(1-q)} \qquad \pi_{2_{\mu=1}}=\frac{p[r_1q+r_2(1-q)]}{r_1pq+r_2(1-q)} \quad [76]$$

The expression for the utility function would be:

$$OF_{(0<\mu_2<1)}=\rho_2\frac{(k+c_1)^{1-\gamma}}{1-\gamma}+(1-\rho_2)\left[\frac{(k+c_2)^{1-\gamma}}{1-\gamma}\pi_{1_{\mu=1}}+\frac{\left(k+c_2\frac{Rl}{Rh}\right)^{1-\gamma}}{1-\gamma}\pi_{2_{\mu=1}}\right]$$
$$c_1=c_{12}+\mu_2(c_{11}-c_{12})$$
$$c_2=c_{21}+(1-\mu_2)(c_{22}-c_{21}) \quad [77]$$

(iii) Finally, if $\mu_2=0$, there is confusion among states 3, 4 and 5[11], then the conditional probabilities would be $\pi_{1_{\mu=0}}$ and $\pi_{2_{\mu=0}}$ and the utility function:

$$OF_{(\mu_2=0)}=\rho_2\frac{(k+c_{12})^{1-\gamma}}{1-\gamma}+(1-\rho_2)\left[\frac{(k+c_{22})^{1-\gamma}}{1-\gamma}\pi_{1_{\mu=0}}+\frac{\left(k+c_{22}\frac{Rl}{Rh}\right)^{1-\gamma}}{1-\gamma}\pi_{2_{\mu=0}}\right] \quad [78]$$

First remark:

$$\lim_{\mu\to 1} OF_{0<\mu<1}=OF_{\mu=1} \quad [79]$$

that is,

$$\lim_{\mu\to 1} OF_{(0<\mu_2<1)}=\rho_2\frac{(k+c_{11})^{1-\gamma}}{1-\gamma}+\frac{1-\rho_2}{1-\gamma} \quad [80]$$

The utility function is continuous for $\mu_2>0$[12], and hence in order to assure a maximum at $\mu_2^*=1$, the following condition(s) should hold:

$$\rho_2\frac{(k+c_{11})^{1-\gamma}}{1-\gamma}+(1-\rho_2)\left[\frac{(k+c_{21})^{1-\gamma}}{1-\gamma}\pi_{1_{\mu=1}}+\frac{\left(k+c_{21}\frac{Rl}{Rh}\right)^{1-\gamma}}{1-\gamma}\pi_{2_{\mu=1}}\right]>$$
$$>\rho_2\frac{(k+c_{12})^{1-\gamma}}{1-\gamma}+(1-\rho_2)\left[\frac{(k+c_{22})^{1-\gamma}}{1-\gamma}\pi_{1_{\mu=0}}+\frac{\left(k+c_{22}\frac{Rl}{Rh}\right)^{1-\gamma}}{1-\gamma}\pi_{2_{\mu=0}}\right] \quad [81]$$

$$\frac{\partial OF}{\partial\mu_2}=\rho_2(k+c_1)^{-\gamma}(c_{11}-c_{12})-(1-\rho_2)(k+c_2)^{-\gamma}\pi_{2_{\mu=1}}>0 \quad [82]$$

The first of these conditions is only needed when the utility function is discontinous in $\mu_2=0$.

In order to assure that the second condition holds, it is only necessary to look for the extreme value of q for which the following condition is satisfied:

$$\left.\frac{\partial OF}{\partial \mu_2}\right|_{\mu_2^*=1} = \rho_2 (k + c_{11})^{-\gamma} (c_{11} - c_{12}) - (1 - \rho_2)(k + c_{21})^{-\gamma} \pi_{2_{\mu=1}} = 0 \qquad [83]$$

Finally, with respect to state 6, there are two different expressions for the utility function:

(i) For any $0 \le \mu_2 < 1$, state 6 is never confounded with any other state and so the uninformed type-2 agents assign probability one to being in state 6. The utility function is given by:

$$OF_{(0 \le \mu_2 < 1)} = \rho_2 \frac{(k+c_1)^{1-\gamma}}{1-\gamma} + (1-\rho_2) \frac{\left(k+c_2 \frac{Rl}{Rh}\right)^{1-\gamma}}{1-\gamma} \qquad [84]$$

$$c_1 = c_{12} + \mu_2 (c_{11} - c_{12})$$

$$c_2 = c_{21} + (1 - \mu_2)(c_{22} - c_{21})$$

(ii) If $\mu_2 = 1$, the utility function coincides with that of states 3 and 4, that is given in equation [75].

It can be shown, as before, the continuity of the utility function, and so we just need to impose that:

$$\frac{\partial OF}{\partial \mu_2} = \rho_2 (k+c_1)^{-\gamma} (c_{11} - c_{12}) > 0 \qquad [85]$$

and this condition is always satisfied.

References

1.- Akerlof, G. 1970, The market for Lemons, Quality Uncertainty and the Market Mechanism, Q.J.E 84, 488-500.

2.- Bhattacharya, S. and C. Jacklin, 1988, Distinguishing Panics and Information-based Bank Runs: Welfare and Policy Implications, Journal of Political Economy 96, 568-592.

3.- Bhattacharya, S. and D. Gale, 1987, Preference shocks, Liquidity and Central Bank Policy, New Approaches to Monetary Economics, WA, Barnett and K.J Singletons (eds). Cambridge University Press, 69-88.

4.- Bhattacharya, S. 1982, Aspects of Monetary and Banking Theory and Moral Hazard, Journal of Finance 37, 374-384.

5.- Bhattacharya, S. 1994, The Economics of Bank Regulation, Université Catholique de Louvain.

6.- Bhattacharya, S. and A.V. Thakor, 1995, Contemporary Banking Theory, Journal of Financial Intermediation 3, 2-50.

7.- Boot, A.W. and S. Greenbaum, 1992, Bank regulation, reputation and rents, Capital Markets and Financial Intermediation, Edited by Mayer, C. and X. Vives, 262-285.

8.- Bryant, J. 1980, A Model of Reserves, Bank Runs and Deposit Insurance, Journal of Banking and Finance 4, 335-344.

9.- Chari, V. and R. Jagannathan, 1988, Banking Panics, Information and Rational Expectations Equilibrium, Journal of Finance 43(3), 749-761.

10.- Craine, R. 1995, Fairly Priced Deposit Insurance and Bank Charter Policy, Journal of Finance 5, 1735-1746.

11.- Dewatripont, M. and J. Tirole, 1994, The Prudential Regulation of Banks, MIT press, Cambridge.

12.- Diamond, D. and P. Dybvig, 1983, Bank Runs, Deposit Insurance and Liquidity, Journal of Political Economy 91(3), 401-419.

13.- Diamond, D. and P. Dybvig, 1986, Banking Theory, Deposit Insurance and Bank Regulation, Journal of Business 59, 55-67.

14.- Fama, E. 1980, Banking in the Theory of Finance, Journal of Monetary Economics 6, 39-57.

15.- Friedman, M. and A. Schwartz, 1963, A Monetary History of the United States, 1867-1960, Princeton, N.J, Princeton University Press.

16.- Gorton, G. 1985, Bank Suspension of Convertibility, Journal of Monetary Economics 15, 177-193.

17.- Gorton, G. 1988, Banking panics and business cycles, Oxford University Press.

18.- Gurley, J. and E. Shaw, 1960, Money in the theory of finance, Washington: Brookings Inst.

19.- Jacklin, C. 1987, Demand Deposits, Trading Restrictions and Risk Sharing, E.C Prescott and N.Wallace (eds.), Contractual Arrangements for Intertemporal Trade, University of Minnesota Press 26-47.

20.- Jacklin, C. 1993, Market Rate Versus Fixed Rate Demand Deposits, Journal of Monetary Economics 32, 237-258.

21.- Kareken, J. 1986, Federal Bank Regulatory Policy: A Description and Some Observations, Journal of Business 59, 3-48.

22.- Kareken, J. and J. Wallace, 1978, Deposit Insurance and Bank Regulation: A partial equilibrium exposition, Journal of Business 51, 413-438.

23.- Lewis, A. and G. Pescetto, 1996, EU and US banking in the 1990s, Academic Press Limited.

24.- Stiglitz, J. 1974, Information and capital markets, Manuscript. Oxford University.

25.- Tobin, J. 1965, The theory of portfolio selection, In the Theory of Interest Rates, edited by Frank, H and F.P.R Brechling, London: Macmillan.

Table I.- States of Nature

θ_i i	State t & R	Prob. $p(\theta_i)$	Aggregate consumption at T-1 CT	CT (Prop1 satisfied)
1	t_1 0 $\tilde{R}$	$r_1(1-q)$	$t_1 cc_{11} + (1-t_1) cc_{12}$	-
2	t_1 α Rh	$r_1(1-p)q$	$t_1 cc_{11} + (1-t_1)[\alpha\, cc_{12H} + (1-\alpha) cc_{12}]$	-
3	t_1 α Rl	$r_1 pq$	$t_1 cc_{11} + (1-t_1)[\alpha\, cc_{12L} + (1-\alpha) cc_{12}]$	c_{11}
4	t_2 0 $\tilde{R}$	$r_2(1-q)$	$t_2 cc_{11} + (1-t_2) cc_{12}$	c_{11}
5	t_2 α Rh	$r_2(1-p)q$	$t_2 cc_{11} + (1-t_2)[\alpha\, cc_{12H} + (1-\alpha) cc_{12}]$	-
6	t_2 α Rl	$r_2 pq$	$t_2 cc_{11} + (1-t_2)[\alpha\, cc_{12L} + (1-\alpha) cc_{12}]$	c_{11}

Table II.- Numerical data

Values of the parameters									
t_1	t_2	r_1	r_2	α	q	p	ρ_1	ρ_2	s
0.30	0.51	0.99	0.01	0.30	0.90	0.10	1.00	0.50	0.18

Fixed values of γ	0.80	1.29	1.78
Fixed values of the standard deviation (σ)	0.333	0.302	0.271
Associated value of R_h	1.200	1.189	1.178
Associated value of R_l	0.000	0.100	1.200
Mean $\overline{R}$	1.080	1.080	1.080

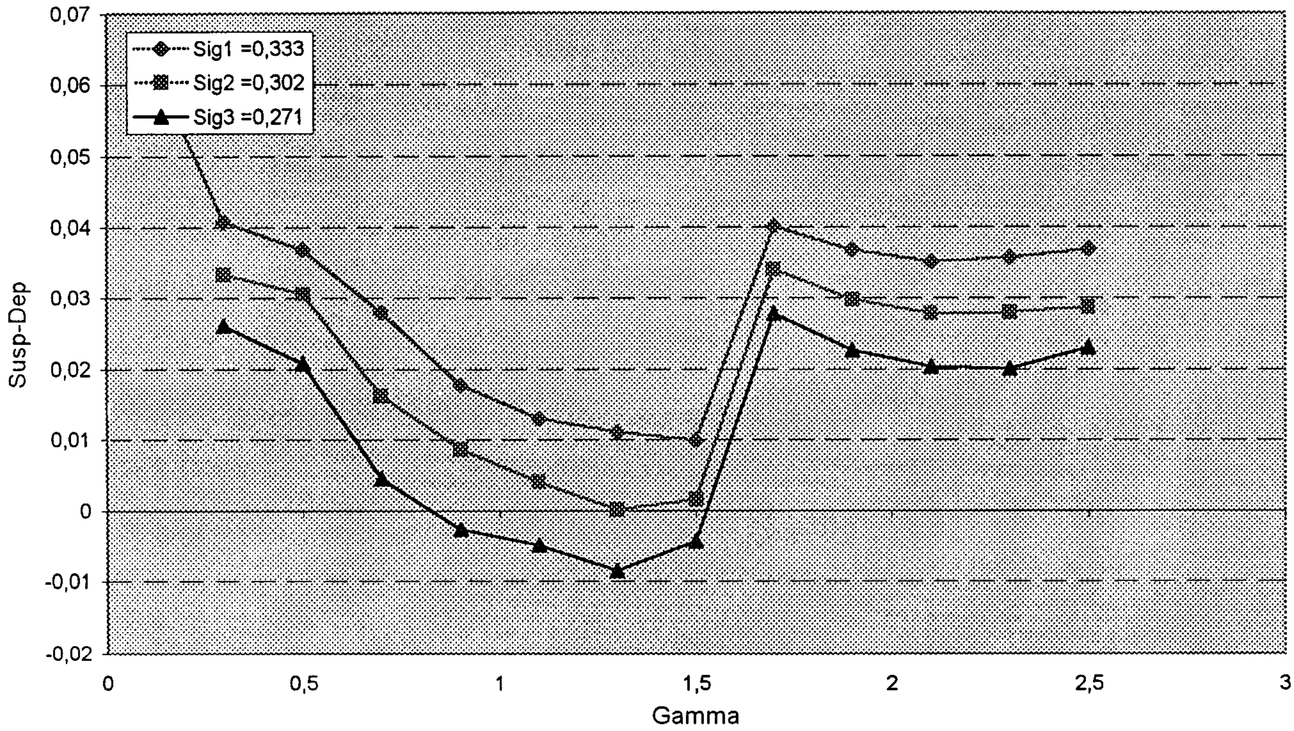

Figure 1.- Suspension minus deposit as a function of γ

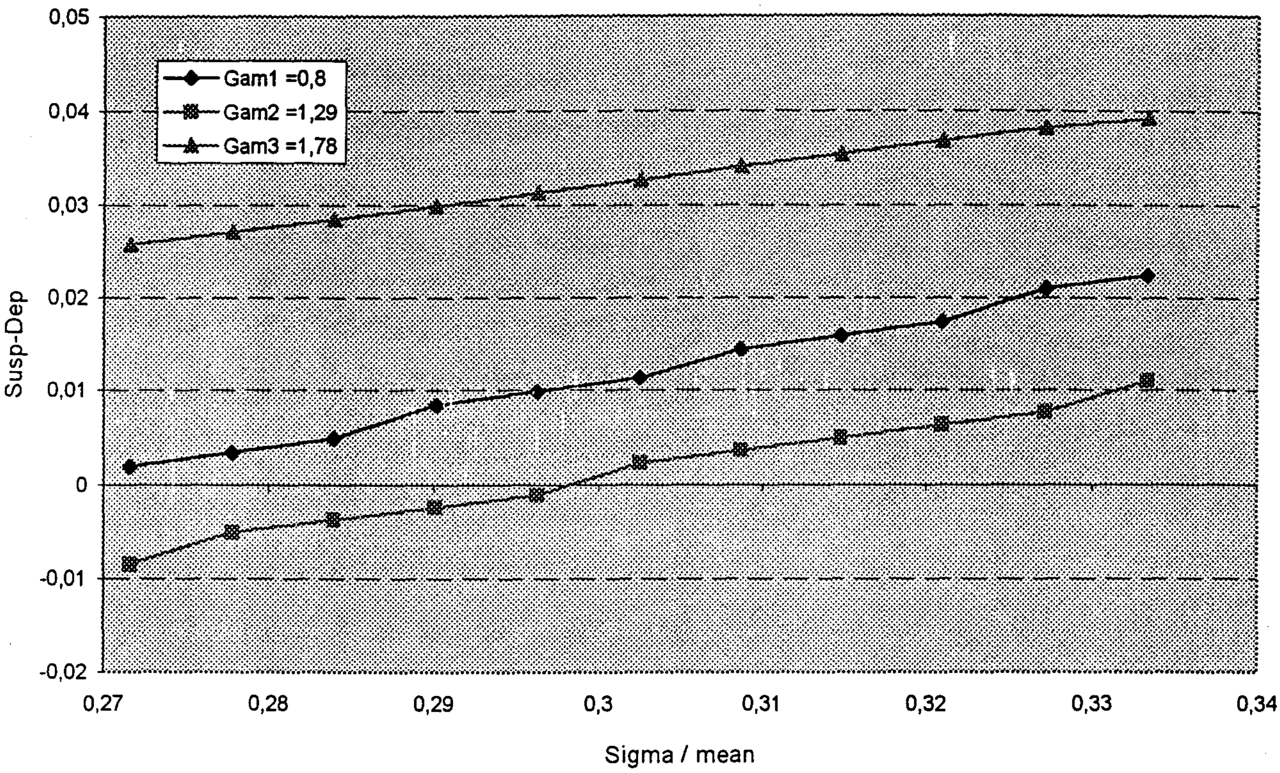

Figure 2.- Suspension minus deposit as a function of σ/mean

Notes

1. *In the model, panic runs may occur due to the fact that uninformed individuals condition their beliefs about the bank's long-term technology on the size of the withdrawal queue at the bank. If this size is large (due to a high liquidity shock only) they may nevertheless infer sufficiently adverse information to precipitate a bank run.*

2. *One interpretation of this assumption is that the asset represents long-term loans which cannot be "called in" early and for which no secondary market exists perhaps due to a "lemmons" problem, as in Akerlof [1].*

3. *This assumption, as in Bhattacharya and Jacklin [2], is motivated by the fact that, if information were costly, type-2 agents would be more likely to purchase the information, and also if depositors were of different sizes, larger depositors would also be more likely to purchase the information. These unmodeled aspects are taken into account, by considering that a random proportion α of type-2 agents becomes informed.*

4. *This function solves the problem of zero consumption having an infinite negative value in terms of utility, when γ is greater than one.*

5. *Formally, this represents the choice for the depositor of* $\mu\,(0 \le \mu \le 1)$*:*

$\mu=1$ *implies the choice of the type-1 contract* $(c_{11}, \bar{c}_{21})$

$\mu=0$ *implies the choice of the type-2 contract* $(c_{12}, \bar{c}_{22})$

$0<\mu<1$ *implies a combination of the two contracts*

6. *It is assumed that whenever the realized* $\tilde{t}$ *is greater than the ex-ante one (average* t *), the bank receives a subsidy from the government in order to cover withdrawals up to the highest proportion of early diers. It could be seen as the government acting as lender-of-last-resort.*

7. *The assumption* $Rl \to 0$ *has been relaxed, although sufficiently small values for Rl have been considered so that bad information about asset quality leads always to a run. See Appendix A.1 for the calculation procedure in this case.*

8. *This result may be due to the fact that in these type of utility functions* γ *represents the relative risk-aversion coefficient and* $1/\gamma$ *the intertemporal elasticity of substitution. A next step is to consider the "isoleastic" CES specification that separates intertemporal substitution and insurance effects.*

9. *This simplification is justified by the "first-come-first-served" nature of the deposit contract.*

10. *See Jacklin and Bhattacharya [2] for a proof of this result.*

11. *This confusion only occurs if informed type-2 agents have also chosen* $\mu_1^* = 0$ *in state 5, otherwise, if* $\mu_2 = 0$ *there is only confusion between states 3 and 4 and therefore the conditional probabilities and the utility function would coincide with those expressed in point 2).*

12. *Or the utility function may be continuous in the interval [0,1] whenever points 2) and 3) coincide, as explained in footnote 11.*